

Subject SP8

CMP Upgrade 2024/25

CMP Upgrade

This CMP Upgrade lists the changes to the Syllabus, Core Reading and the ActEd material since last year that might realistically affect your chance of success in the exam. It is produced so that you can manually amend your 2024 CMP to make it suitable for study for the 2025 exams. It includes replacement pages and additional pages where appropriate.

Alternatively, you can buy a full set of up-to-date Course Notes / CMP at a significantly reduced price if you have previously bought the full-price Course Notes / CMP in this subject. Please see our *2025 Student Brochure* for more details.

We only accept the current version of assignments for marking, *ie* those published for the sessions leading to the 2025 exams. If you wish to submit your scripts for marking but only have an old version, then you can order the current assignments free of charge if you have purchased the same assignments in the same subject in a previous year, and have purchased marking for the 2025 session.

This CMP Upgrade contains:

- all significant changes to the Syllabus and Core Reading
- additional changes to the ActEd Course Notes and Assignments that will make them suitable for study for the 2025 exams.

0 Changes to the Syllabus

There have been no changes to the syllabus objectives.

1 Changes to the Core Reading and ActEd material

This section contains all the *non-trivial* changes to the Core Reading and ActEd text.

Chapter 3

Section 1.3

The section entitled 'Product liability' has been amended and now reads:

Policies are likely to be written on a claims-made basis.

However, it can also be written on a losses-occurring basis.

Chapter 6

Section 7.3

Under the heading 'Enhancing the creditworthiness of debt instruments / providing capital relief to banks', the last sentence of the second paragraph has been amended and now reads:

In order to do this, it must either securitise its best quality loans (*ie* the loans with the lowest risk of default), or it must securitise a large number of loans relative to the number of bonds issued (in which case, even if some of the loans default, there will still be an adequate number of performing loans left with which to make payments on the bonds).

Chapter 7

Section 3.2

In the final sentence of the fourth bullet point, the word 'European' has been replaced with 'EU'.

Chapter 8

Section 1.3

The following paragraph has been added at end of the subsection with heading 'Simplistic capital regimes':

Following Brexit, the UK insurance market is moving from Solvency II rules to 'Solvency UK' rules (abbreviated as SUK). The intention is that SUK should be simpler and more flexible than Solvency II. The implementation date for Solvency UK is 31st December 2024.

Chapter 10

At the end of Section 5.1, the following text has been added:

The term 'heads of damage' is more commonly used to refer to the different components of injury claims, such as compensation for:

- loss of income
- medical and nursing costs
- pain and suffering.

Fire, theft, explosion, *etc* are more commonly referred to as perils, or indeed claim types.

Appendix: Formulae

A new appendix that acts as a quick-reference guide to some of the key formulae in Subject SP8 has been added to the Course Notes. This appendix is included at the end of this upgrade document.

2 Changes to the X Assignments

Overall

The X Assignments have been changed significantly. We have not detailed all of the changes in this upgrade.

If you would like the new assignments *without* marking, then retakers can purchase an updated CMP or standalone X Assignments at a significantly reduced price. Further information on retaker discounts can be found at:

acted.co.uk/paper_reduced_prices.html

If you wish to submit your scripts for marking but only have an old version, then you can order the current assignments free of charge if you have purchased the same assignments in the same subject in a previous year, and have purchased marking for the 2025 session. We only accept the current version of assignments for marking, *ie* those published for the sessions leading to the 2025 exams.

3 Changes to the Mock Exam

Overall

The Mock Exam has been changed significantly. We have not detailed all of the changes in this upgrade.

If you would like the new Mock Exam *without* marking, then retakers can purchase an updated CMP or standalone Mock Exam at a significantly reduced price. Further information on retaker discounts can be found at:

acted.co.uk/paper_reduced_prices.html

If you wish to submit your scripts for marking but only have an old version, then you can order the current Mock Exam free of charge if you have purchased the same Mock Exam in the same subject in a previous year, and have purchased marking for the 2025 session. We only accept the current version of the Mock Exam for marking, *ie* those published for the sessions leading to the 2025 exams.

4 Other tuition services

In addition to the CMP, you might find the following services helpful with your study.

4.1 Study material

For further details on ActEd's study materials, please refer to the *Products* pages on the ActEd website at **ActEd.co.uk**.

4.2 Tutorials

We offer the following (face-to-face and/or online) tutorials in Subject SP8:

- a set of Regular Tutorials (lasting a total of three days)
- a Block (or Split Block) Tutorial (lasting three full days)
- an Online Classroom.

For further details on ActEd's tutorials, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **ActEd.co.uk**.

4.3 Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

For further details on ActEd's marking services, please refer to the *2025 Student Brochure*, which is available from the ActEd website at **ActEd.co.uk**.

4.4 Feedback on the study material

ActEd is always pleased to receive feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments (*eg* about certain sections of the notes or particular questions) or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course, please send them by email to **SP8@bpp.com**.

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Appendix

Formulae

0 Introduction

This appendix is a useful quick-reference guide to some of the key formulae in Subject SP8 – it is not intended to be exhaustive.

1 Formulae

1.1 Reinsurance

Surplus reinsurance

- Max size of risk that can be placed on a surplus treaty: $(1 + L) \times R$
- For a specific risk, the number of lines ceded is: $(E - r) / r$
- For a specific risk: $E = r \times (k + 1)$

where:

E = estimated maximum loss

L = number of lines of cover available R

k = number of lines chosen

R = maximum retention

r = chosen retention.

Non-proportional reinsurance

Rate on line = $\frac{\text{reinsurance premium charged (ignoring reinstatement premiums)}}{\text{width of layer covered}}$

1.2 Aggregate claim distribution models

Collective risk model

The mean and variance of S under the collective risk model are given by:

- $E(S) = E(N)E(X)$
- $\text{var}(S) = E(N)\text{var}(X) + \text{var}(N)[E(X)]^2$.

The moment generating function of S under the collective risk model is given by:

$$M_S(t) = M_N[\log M_X(t)].$$

Panjer's recursive formula

If claim amounts take discrete values and there exist a and b such that:

$$P(N=n) = \left(a + \frac{b}{n}\right)P(N=n-1), \quad n=1, 2, \dots$$

where N denotes the claim number random variable, then the following recursive formula can be used to calculate the probability function of the aggregate claim amount random variable S :

$$P(S=0) = P(N=0)$$

$$P(S=s) = \sum_{x=1}^s \left(a + \frac{bx}{s}\right)P(X=x)P(S=s-x), \quad s=1, 2, \dots$$

Translated gamma approximation

If we know the mean, variance and skewness of S , we can approximate the distribution of S using a translated gamma distribution $S \sim \text{Gamma}(\alpha, \delta) + k$. The values of k , α and δ can be calculated using the equations:

$$E(S) = \frac{\alpha}{\delta} + k$$

$$\text{var}(S) = \frac{\alpha}{\delta^2}$$

$$\text{skew}(S) = \frac{2\alpha}{\delta^3}$$

In place of the third equation given above, we could use the fact that:

$$\text{Coefficient of skewness} = \frac{2}{\sqrt{\alpha}}$$

Gamma probabilities are not listed in the *Tables*. However, they can be calculated using the relationship between the gamma and chi-squared distributions: $Y \sim \text{Gamma}(\alpha, \delta) \Leftrightarrow 2\delta Y \sim \chi_{2\alpha}^2$

1.3 Premium rating

The relevant equation of value is:

$$\text{PV of gross premium} = \text{PV of claims costs} + \text{PV of expenses and loadings}$$

Burning cost approach

The burning cost premium is given by: $(\sum \text{claims}) / (\text{total exposed to risk})$.

Frequency-severity approach

For each historical policy year, we can calculate the frequency of losses as:

$$\text{average frequency} = \frac{\text{ultimate number of losses}}{\text{total exposure}}$$

For each historical policy year, we can calculate the average severity of losses as:

$$\text{average severity} = \frac{\text{ultimate cost of losses}}{\text{ultimate number of losses}}$$

1.4 Original loss curves

$$LEV_X(x) = \int_0^x S_X(y) dy \quad \text{and} \quad LEV'_X(x) = S_X(x)$$

where $S_X(x)$ is the survival function, ie $S_X(x) = 1 - F_X(x)$.

Property business

$$C_L = \text{Layer loss cost} = C \times \left(G\left(\frac{L+D}{M}\right) - G\left(\frac{D}{M}\right) \right)$$

or if there is an underlying deductible d , we use the formula:

$$C_L = \text{Layer loss cost} = C \times \frac{\left(G\left(\frac{L+D+d}{M+d}\right) - G\left(\frac{D+d}{M+d}\right) \right)}{\left(1 - G\left(\frac{d}{M+d}\right) \right)}$$

Liability business

$$ILF(x) = \frac{LEV_X(x)}{LEV_X(b)}, \quad \text{where the base limit is } b.$$

$$C_L = \text{Layer loss cost} = C_I \times \left(\frac{ILF(L+D) - ILF(D)}{ILF(I)} \right), \quad \text{where } I \text{ is the original limit.}$$

For base level b , if we expect losses to have increased by $a\%$ between time t and time t' :

$$ILF_{t'}(x) = \frac{ILF_t\left(\frac{x}{1+a}\right)}{ILF_t\left(\frac{b}{1+a}\right)}$$

1.5 Generalised linear models

Exponential family

For distributions in the exponential family:

- the probability function or probability density function can be written in the form:

$$f_i(y_i; \theta_i, \varphi) = \exp\left\{\frac{y_i\theta_i - b(\theta_i)}{a_i(\varphi)} + c(y_i, \varphi)\right\}$$

- the mean and variance of Y_i satisfy the equations:

$$E(Y_i) = b'(\theta_i)$$

$$\text{var}(Y_i) = a_i(\varphi)b''(\theta_i)$$

- the variance function is given by:

$$V(\mu) = b''(\theta)$$

The mean and variance functions for some common distributions:

Distribution	$\mu(\theta)$	$V(\mu)$
Normal	θ	1
Poisson	e^θ	μ
Gamma	$-1/\theta$	μ^2
Binomial	$e^\theta / (1 + e^\theta)$	$\mu(1 - \mu)$
Exponential	$-1/\theta$	μ^2

Structure

A GLM has the following structure: $Y_i = g^{-1} \left(\sum_{j=1}^k X_{ij} \beta_j + \xi_i \right) + \varepsilon_i$

or in matrix form: $\mathbf{Y} = g^{-1} (\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\xi}) + \boldsymbol{\varepsilon}$.

where:

- \mathbf{X} is the design matrix of factors
- $\boldsymbol{\beta}$ is a vector of parameters to be estimated
- $\boldsymbol{\xi}$ is a vector of offsets or known effects
- $\boldsymbol{\varepsilon}$ is the error term appropriate to \mathbf{Y} .

The linear predictor is $\boldsymbol{\eta} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\xi}$.

The logit link function is $g(y) = \text{logit}(y) = \ln \left(\frac{y}{1-y} \right)$.

A summary of typical model forms:

Model	Error distribution	Link function	Scale parameter φ	Variance function $V(\mu)$	Weights ω_i	Offset ξ_i
Claim frequency	Poisson	$\ln(y)$	1	μ	Exposure	0
Claim numbers	Poisson	$\ln(y)$	1	μ	1	$\ln(\text{Exposure})$
Claim severity	Gamma	$\ln(y)$	Estimated	μ^2	Claim numbers	0
Total claim cost	Tweedie	$\ln(y)$	Estimated	$\mu^{1.5}$	Exposure	0
Propensity	Binomial	$\ln \left(\frac{y}{1-y} \right)$	1	$\mu(1-\mu)$	1	0

Testing model appropriateness

The scaled deviance is $D^* = \frac{D}{\varphi}$, where D is the deviance and φ the scale parameter.

We can compare the goodness of fit of nested models 1 and 2 using:

$$D_1^* - D_2^* \sim \chi_{df_1 - df_2}^2$$

If φ is unknown, the goodness of fit of nested models can be compared using:

$$\frac{(D_1 - D_2)}{(df_1 - df_2)(D_2 / df_2)} \sim F_{df_1 - df_2, df_2}$$

If models are not nested, we can use: $AIC = -2 \times \log\text{-likelihood} + 2 \times \text{number of parameters}$

The i th Pearson residual is:

$$r_i^P = \frac{Y_i - \mu_i}{\sqrt{\frac{\varphi V(\mu_i)(1 - h_{ii})}{\omega_i}}} = \frac{Y_i - E[Y_i]}{SD[Y_i] \sqrt{1 - h_{ii}}}$$

The i th deviance residual is:

$$r_i^D = \text{sign}(Y_i - \mu_i) \sqrt{d(Y_i; \mu_i)}$$

where:

$$d(Y_i; \mu_i) = 2\omega_i \int_{\mu_i}^{Y_i} \frac{(Y_i - \zeta)}{V(\zeta)} d\zeta$$

Cook's distance for the i th data point is given by:

$$c_i^P = \frac{h_{ii}}{(1 - h_{ii}) \left(\sum_i h_{ii} \right)} (r_i^P)^2$$

Cramer's V statistic:
$$\sqrt{\frac{\sum_{i,j} \frac{(n_{ij} - e_{ij})^2}{e_{ij}}}{\min((a-1), (b-1)) \cdot n}}$$

1.6 Credibility theory

Credibility weighted estimate = $Z \times \text{Observation} + (1 - Z) \times \text{Other information}$, $0 \leq Z \leq 1$

Standards for full credibility in classical credibility theory

If the goal is to be within a proportion $\pm k$ of the mean μ , with a probability of at least P , then for $\Phi(y) = \frac{1+P}{2}$, the standard for full credibility is:

$$\text{Frequency (Poisson)} \quad n_N = \frac{y^2}{k^2}$$

$$\text{Frequency (general)} \quad n_N = \frac{y^2}{k^2} \left(\frac{\sigma_N^2}{\mu_N} \right)$$

$$\text{Severity} \quad n_X = \frac{y^2}{k^2} CV_X^2 \quad \text{where } CV_X = \sigma_X / \mu_X, \text{ the coefficient of variation of the claim size}$$

$$\text{Aggregate losses (Poisson frequency)} \quad n_S = n_N + n_X$$

$$\text{Aggregate losses (general)} \quad n_S = \left(\frac{y}{k} \right)^2 \left(\frac{\sigma_N^2}{\mu_N} + \frac{\sigma_X^2}{\mu_X^2} \right)$$

Partial credibility

If n is the (expected) number of claims for the volume of data and n_N is the standard for full credibility, then for $n > n_N$, $Z = 1$ and for $n \leq n_N$:

$$\text{Classical formula (square root rule)} \quad Z_C = \left(\frac{n}{n_N} \right)^{1/2}$$

$$\text{Bayesian formula} \quad Z_B = \frac{n}{n+k} \quad (\text{expressed in units of claims}).$$

Bühlmann-Straub model

S_i represents the insurance claims for risk i

V_i represents the volume measure for risk i

$X_i = S_i / V_i$ represents the claims ratio for risk i

$$\mu(\theta_i) = E(X_i | \theta_i) \quad \sigma^2(\theta_i) = V_i \times \text{Var}(X_i | \theta_i)$$

$$\beta = E[\mu(\theta_i)] \quad \phi = E[\sigma^2(\theta_i)] \quad \lambda = \text{Var}[\mu(\theta_i)]$$

Credibility factor:
$$z_i = \frac{V_i}{V_i + \phi/\lambda}$$

Credibility premium:
$$C^{BE} = z_i X_i + (1 - z_i) \beta$$

1.7 Actuarial investigations

The rate change for a group of renewed policies can be expressed as:

$$\text{Rate Change}_{t_1 \rightarrow t_2} = \frac{\sum \text{Prem}_{t_2}}{\sum \text{As-if Prem}_{t_1}} - 1$$

$$\text{Lapse rate} = \frac{\text{no. of lapses for period } x}{\text{no. of renewals invited for period } x}$$

$$\text{New business rate} = \frac{\text{no. of new policies for period } x}{\text{no. of renewals invited for period } x}$$

$$\text{Endorsement rate} = \frac{\text{no. of endorsements during period } x}{\text{no. of policies exposed for period } x}$$

$$\text{Cancellation rate} = \frac{\text{no. of cancellations during period } x}{\text{no. of policies exposed for period } x}$$

or, as an approximation:

$$\text{Cancellation rate} = \frac{\text{no. of cancellations during period } x}{\text{no. of renewals invited for period } x}$$

$$\text{Renewal rate} = \frac{\text{no. of renewals for period } x}{\text{no. of expiring policies in period } x}$$

or

$$\text{Renewal rate} = \frac{\text{no. of renewals for period } x}{\text{no. of renewals invited for period } x}$$